7 EFFICIENT STRATEGIES
An Application in Water Quality Planning

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1. Introduction

New information technologies like multimedia, virtual reality, and geographic information systems (GISs) within the framework of decision support systems (DSSs) present planners with new and exciting opportunities. These include enhanced DSS efficiency through the display of graphic information, which allows rapid, integrated assessment of one or more decision alternatives. This chapter describes a graphics-based decision support technique that provides information on the outcomes of a very large (or infinite) variety of possible decision strategies, and helps the decision-maker select the best option. This technique directly supports the two steps of the decision choice phase, as shown in Figure 1. By means of decision screening, this technique selects a small number of strategies and subjects them to further detailed exploration and what-if analysis. It has both static and dynamic graphic components.
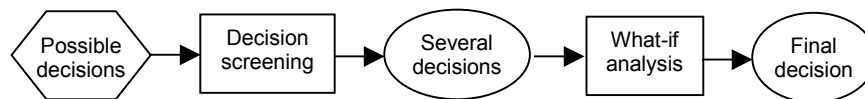

Figure 1. Decision choice steps.

Section 2 introduces the applications of this technique in support of water quality management. Section 3 describes some key environmental problems and discusses the usefulness of simplified integrated models for decision screening. Section 4 introduces the feasible goal method (FGM) and interactive decision map (IDM) techniques. Section 5 outlines the methodology of the search for effective environmental strategies. In Section 6, the DSS developed for Russia's Resurrection of the Volga

River program is used as a case study to illustrate both the methodology and the FGM/IDM technique. Section 7 describes Web-based applications of the technique and the Appendix provides a mathematical description.

2. Environmental decisions

The applications of this decision support technique are based on a methodology developed for an integrated assessment of environmental problems. The methodology described is applied in a DSS for water quality planning in river basins, and is used to help provide efficient and effective solutions for water quality problems.

Many computer-based DSSs provide what-if analysis for only a few decision alternatives. Unfortunately, environmental problems often have a very large (or infinite) number of alternative solutions. This means that the decision-maker has the complicated task of selecting only a few alternatives for the what-if analysis. Experts usually are asked to develop several decision alternatives so that decision-makers need not face the perhaps thousands of options to arrive at their decision. The strategies experts suggest will reflect their experience, perceptions, and goals, which may well differ from those of the decision-makers. This can result in deadlock during the implementation phase, especially if decision-makers are forced to choose from among strategies which do not reflect their opinions or interests.

The importance of decision screening in water management problems has been stressed by Dorfman (1965), who articulated the need for simplified models. Often, single-criterion optimization is considered adequate to help to screen the decision alternatives. For environmental problems, however, there is rarely a single criterion situation: decision screening must be based on multiple-criterion methods. Examples of their use in water management are described in Cohon and Marks (1975), Cohon (1978), Loucks *et al.* (1981), Moiseev (1982), and Louie *et al.* (1984).

Certain aspects of environmental problems complicate the development of efficient strategies to resolve them. They are summarized below.

- Environmental problems often involve competing interests of different social groups.

- Environmental decisions are made by politicians who tend to view problems on a case-by-case basis, rather than considering the whole picture. In addition, their personal priorities may sometimes obscure their official objectives.

- Politicians have little time to study a large volume of information on any given topic; they need information presented in a concise and comprehensive way.

- The public must be kept informed about environmental decisions made on their behalf: environmental problems affect everyone.

- Environmental modeling requires the integration of knowledge from a number of other disciplines, including ecology, economics, geography, and the social sciences. Also, the environmental model should include descriptions of its subsystems, including treatment of pollutants, transport of pollutants, and the effect of

pollutants on the ecology, as well as the economic impacts, environmental measures, and so forth.

The methodology presented in this chapter satisfies all of these requirements. Introduced in 1994 (Lotov, 1994; Lotov *et al*., 1997b; Lotov, 1998), it is based on two main principles:

1. strategy options are screened through the application of integrated mathematical models that combine simplified mathematical descriptions, expert judgements, and empirical data; and

2. strategy options are screened by a user by means of the FGM and IDM techniques that support the exploration of efficient frontiers among aggregated economic and environmental choice criteria, and the identification of a feasible goal.

3. Integrated environmental problem models

The application of simplified integrated mathematical models to screen various strategy options is an important part of the methodology described in this paper. An integrated mathematical model combines simplified mathematical descriptions of the subsystems with expert judgments and empirical data. An integrated model is needed because the original mathematical model that describes the subsystems of a particular environmental problem (for example, discharges of pollutants, treatment of pollutants, transport of pollutants, impacts on the ecosystem, health impacts) may be diverse, and usually has been constructed by specialists from different fields. Since every scientific field has its own mathematical language, combining, or even coordinating these models is problematic. Hence the need to simplify and integrate the models, and sometimes supplement them with expert judgments and empirical data.

3.1. Constructing simplified integrated models

The most universal way to build an integrated model is to establish an approximation for the input-output dependencies, or parameterization, of the original model. Consider a stationary linear model describing the regional transport of a single pollutant from several pollution sources. Assume that a linear model of partial derivatives can be used to evaluate the pollutant concentration at any point in the region and that the pollution discharge per unit time is constant. If formulated correctly, the problem of determining the concentration of the single pollutant is solved.

If pollutant discharge data are not known, the point source method can be used; source functions describing the concentrations that result from the individual source and the unit rate of discharge are constructed. Since the pollutant transport model is linear, the concentration at any given point (for example, at a monitoring point) may be calculated as the sum of the products of the source function values for real discharges. In other words, the concentration of the pollutant at this point is a linear function of pollution discharges, denoted by the scalar product of the influence coefficient vector and the discharge vector.

Given the finite number of monitoring points where the pollutant concentrations are of interest, concentration values can be calculated by multiplying the influence matrix (its rows are provided by influence coefficient values) by the discharge vector. In the linear case, influence matrices can be calculated precisely by using the values of the source functions. In the nonlinear case, influence matrices can only approximate the dependencies of the pollution concentrations on the discharges; how they are constructed depends upon the particular scientific field. The universal approach is based on the application of regression analysis of data on input-output dependencies, obtained by the simulation of nonlinear models. Along with the approximation of an influence matrix, the simulation provides applicability ranges for its elements.

It is often impossible to construct a single influence matrix whose applicability range covers all possible inputs. In this case, several influence matrices may be developed. Each matrix is related to a certain applicability range; all of the matrices cover all possible inputs. Note that within the framework of environmental issues, an applicability range can describe a variety of input values that do not change the environmental system qualitatively. If the limits of this kind of applicability range are violated, it may not be possible for the system to exist in the same qualitative form.

If there is no adequate mathematical model for a subsystem, an influence matrix can be constructed through regression analysis of experimental or historical data. Sometimes, experts can provide both an influence matrix and its applicability range. This chapter does not describe other forms of simplified models as they are not universal and are often based on specific features of the particular system.

A combination of influence matrices and other simplified descriptions, balance equations, and restrictions imposed on variables contributes to an integrated model that describes an environmental system. Simplified integrated models are typically less precise then the original models; however, precision is not of paramount importance, as integrated models are used only to screen feasible strategies. Consequently, the rough description offered by a simplified integrated model may be sufficient to identify a feasible goal. Moreover, it is assumed that any strategy predicated on a simplified model will be verified and improved later, through simulations with adequate subsystem models.

3.2. An example

Simplified integrated models were produced with the DSS used by Russian environmental engineers who had developed reasonable strategies for water quality improvement in small river basins. A detailed description of this system is provided by Lotov and colleagues (1997a).

A stretch of river was divided into a number of segments, with a monitoring station at the downstream end of each segment to monitor the concentrations of several dozen industrial pollutants. Polluting industrial enterprises were grouped by production technology and pollutant output; municipal water services were grouped in the same fashion. About 20 polluting industries and municipal water services were included in the integrated model. The industrial enterprises and municipal services were grouped by river segment, in line with the experts' recommendation. This grouping of the polluting enterprises was an important simplification.

Discharge treatment processes by an industry or a water service also were described in a simplified form. The discharge treatment model describing the decrement in the cost of wastewater treatment for an industry or service depends on the specific wastewater purification technology. The model was based on a database of the discharge treatment technologies, also developed by the experts.

Pollutant transport was described approximately by an influence matrix that made it possible to compute the concentrations of pollutants at the monitoring stations by multiplying the influence matrix by the discharge vector. In general, the coefficients of the influence matrix can be developed through simulation of detailed pollutant transport models; in this case, however, they were estimated by the experts.

4. The FGM/IDM technique: an informal introduction

This section offers an informal introduction of the FGM/IDM technique on the basis of the simplified integrated model outlined in the Introduction. One decision variable of the model describes the volume of discharge from a particular industry or service that is treated with a particular wastewater purification technology in a particular river segment. A collection of such variables constitutes a water quality improvement strategy. Several criteria options can be used in the search for a preferred strategy. This illustration of the FGM/IDM technique incorporates three criteria (see Figure 2): fish stock depletion ("Fishing"), general pollution ("General"), and the cost of the project.
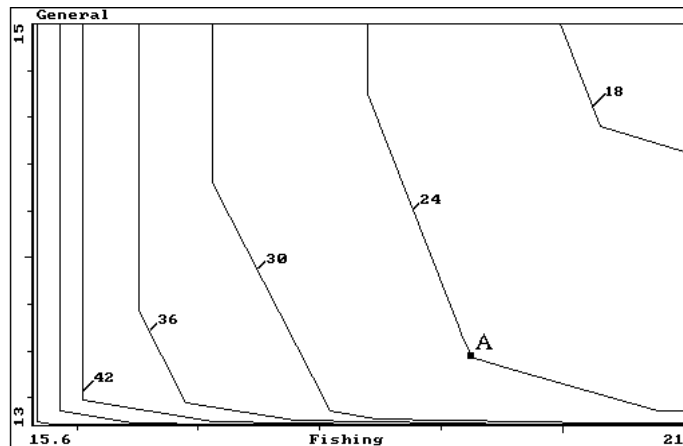

Figure 2. A decision map.

4.1. Interactive decision maps

The FGM/IDM technique is displayed as a decision map which shows the efficiency relationship among the criteria. In this case, the values for the pollution criteria are

represented as "Fishing", or fish stock depletion along the horizontal axis and "General", or general pollution along the vertical axis (Figure 2). The decision map consists of several intervals, each related to a certain cost (indicated near each interval). The cost is given in millions of USSR 1988 rubles.

The interval marked with point A, for example, is related to the cost of 24 million rubles. Any point on the graph (denoting the combination of both criteria) that is located to the right of the interval is feasible or, in other words, indicates a strategy that costs no more than 24 million rubles. Conversely, any point to the left of the interval is not feasible at that cost.

The interval represents the limit of feasible combinations for the two pollution criteria values at the given cost. This limit is of particular interest to decision-makers. It is called the efficient frontier, because its shape shows how much fish stock depletion is related to the increase in general pollution, if the cost is limited to 24 million rubles. Above point A on the interval, relatively small increments of fish stock depletion are needed to decrease the value of general pollution. The slope of the interval changes drastically at A. The efficient frontier thus illustrates the efficient trade-offs between the two pollution criteria for the given cost. It is worth mentioning that the notion of efficiency trade-off clearly differs from the notion of value trade-off used by Keeney and Raiffa (1976); the latter implies subjective compensation of losses in one criterion by gains in the other.

Comparing one efficient frontier to another, one can see how the cost increment affects the number of feasible combinations. For example, if the cost is restricted to 18 million rubles, the value of general pollution cannot be less than about 14.6 and the value of fish stock depletion cannot be less than about 20. Adding six million rubles provides more opportunities to decrease both pollution criteria. For example, at 24 million rubles, the general pollution value of 13.4 becomes feasible. On the other hand, if the cost is increased to 30 million, fish stock depletion can be decreased to a value of about 18.5. Thus, the decision map provides a rough guide on the efficient trade-offs among all three criteria.

A decision map thus plots the efficient frontiers between two criteria, for several values of a third. The FGM/IDM technique applies the concept of a modified decision map (Haimes *et al*., 1990). In the standard decision map technique, several cross-sections of the efficient frontier of the feasible set in criterion space (FSCS) are shown at once. Though the modified decision maps are similar to standard decision maps, they have several advantages – most important, their computing method.

Decision maps are not prepared in advance. First it is necessary to determine an approximation of the Edgeworth-Pareto hull (EPH) of the FSCS. The EPH is a set in the criterion space that, in addition to the feasible criterion values, also contains all dominated criterion values. The algorithm used to obtain the EPH is provided in the Appendix. For linear models, the EPH is a convex set in the criterion space. After the EPH is determined, decision maps are obtained by computing and displaying efficient frontiers (cross-sections) of the EPH. Since the EPH in the criterion space is approximated in advance, several decision maps can be depicted or animated in real-time. Later, references to decision maps will imply the modified version without any additional indication.

Decision maps have several features in common with topographical maps. Since the efficient frontiers of decision maps do not intersect (although they may sometimes

coincide), they resemble the height contours of a topographical map. The value of the third criterion in a decision map is similar to height in a topographical map. Knowledge of topographical maps will aid in the analysis of decision maps. It is easy to view the variety of feasible combinations of the first and second criterion in relation to the value of the third ("places higher than..."). It is also easy to understand which values of the third criterion are feasible for a given combination of the first and second criterion ("height of this point is between..."). The proximity of efficient frontiers in the vicinity of a point indicates a slope, as in a topographical map. A slight shift in the efficient frontier results in a substantial change in the value of the third criterion.

4.2. Feasible goals method

Once the exploration of a decision map is complete, the decision-maker may wish to identify a preferred feasible combination of the criterion values (a feasible goal). This is the main feature of the FGM. If point A (Figure 2) were identified as a feasible goal, the wastewater treatment plan that resulted in the identified values of cost and pollution criteria can be computed automatically (see the Appendix for details). If the IDM technique is applied, a preferred feasible goal can be identified on a decision map with a simple mouse click. Decision-makers do not need to perform complicated interactive procedures to extract their preferences. Methods that are not related to the decision-makers' preferences, but are aimed at the display of the efficient set in criterion space, are named multiple-criterion generation methods (Cohon, 1978). Within this framework, decision-makers are given full freedom of choice with regard to the efficient combinations of the criteria.

The application of multiple-criterion generation methods (including the FGM/IDM technique) may be premised on the position of modern psychology that people base their decisions on their mental models of reality, which – although fairly crude – give an integrated picture of reality (Lomov, 1984).

Not only are mental models often crude, but they are also untrue in many respects. One of the purposes of decision support, therefore, is to enhance a user's capacity to formulate mental models. Knowledge about available options in a clear graphic form can help decision-makers understand it clearly at both the conscious and the subconscious levels. In the context of multiple-criterion generation methods, the FGM/IDM technique depicts knowledge about efficient frontiers for selected criteria in precisely this kind of unambiguous, graphics-based format. The technique does not force users to answer multiple questions about their preferences; rather, it helps decision-makers improve their decisions through a clearer understanding of the situation.

5. Methodology

The main steps of this methodology to find efficient environmental strategies are listed below.
1. Experts conduct an initial qualitative analysis of the problem. They develop a list of subsystems to be considered within the framework of the study, as well as a list of possible criteria (performance indicators).

2. Information on previous studies of the subsystems (including their mathematical modeling) and their interaction is collected. Researchers develop a list of variables that describe the interaction among the subsystems.

3. The ranges of possible variations of the variables are identified.

4. Mathematical models of the subsystems are developed (or adapted) and calibrated. If such models do not exist and cannot be developed in time, experimental and historical data are collected for the statistical analysis of the input-output relations.

5. Influence matrices and other simplified descriptions of the subsystems are developed ,and the applicability ranges are evaluated. If the applicability range of a single influence matrix of a subsystem's model does not contain the ranges of its input variables, the ranges are squeezed or several influence matrices are developed whose applicability ranges cover the ranges of the input variables. The integrated model is constructed by the unification of the influence matrices, and by the imposition of restrictions on the variables.

6. The constraints on possible strategies are specified, allowing the extraction of a set of feasible strategies.

7. Decision-makers select criteria from the list and impose limits on all or some of the performance indicators. The EPH of the FSCS is approximated.

8. Decision-makers explore various decision maps using the IDM software, based on the FGM/IDM technique. From this they gain an understanding of the efficient trade-offs among the criteria. At this point the decision-makers can identify one or several feasible goals; the related strategies then are computed automatically.

9. The most effective and efficient strategies are displayed to the decision-makers. In the case of spatial strategies, GISs may play an important role.

10. Decision-makers select one or more strategies that will provide a starting point for the detailed development of environmental projects. In contrast to the screening stage, the development of environmental projects is based on the simulation of detailed adequate models.

   This is a simplified illustration; in reality, various snags can prevent the process from running smoothly. For example, if a developed strategy violates constraints that had not been included in the model in advance, these constraints may be introduced later, but the imposition will alter the range of feasible strategies. The process would start again at step 6. If the exploration of a strategy shows that a simplified (or even an original) model of a subsystem is imprecise, it would have to return to step 2 and continue the research by developing the subsystem's model.

   This methodology was applied to the development of environmental strategies for ground water management (Kamenev *et al*., 1986; Lotov *et al*., 1997b), atmosphere pollution abatement (Bushenkov *et al*., 1994; Lotov *et al*., 1997b), and global climate change (Lotov, 1994; Lotov *et al*., 1997b).

6. A DSS for water quality planning in large rivers

As noted above, the FGM/IDM technique was implemented in the water quality planning support system developed for the Resurrection of the Volga River program. Pollutant transport data (the influence matrices) were obtained through the use of IRAS simulation software developed by Loucks and colleagues (1985). The technique helped decision-makers identify feasible goals for water quality and devise strategies for investment allocation among multiple regions in the river basin.

6.1. Criteria

The model has several thousand variables. Only 100, however, were considered as possible criteria (performance indicators). These variables were displayed at the very beginning of the study (Figure 3). The approximation technique used in the DSS allows the user to select between two and seven criteria from the hundred. Users also may specify limits on the values of all selected indicators.
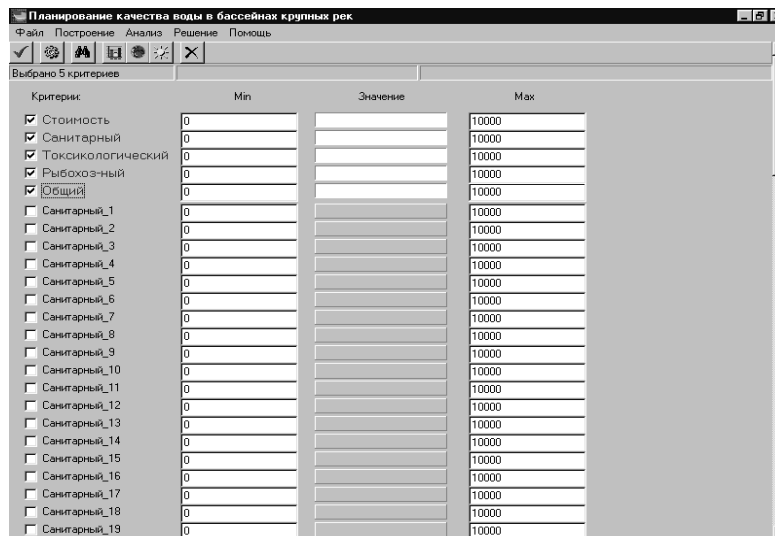

Figure 3. Selection of the first five performance indicators.

In Figure 3, the first five indicators are chosen as criteria and no constraints are imposed on their values. These criteria are the cost of the project and four aggregated water quality criteria. The cost is calculated as the sum of all costs related to the application of particular discharge treatment technologies. The aggregated water quality criteria describe pollution levels at the monitoring stations. As more than twenty pollutants are being tested for, they are grouped into aggregated environmental indicators. The values of these indicators are measured in relative units. The desired value of an aggregated environmental indicator is equal to one. Among monitoring stations,

the maximal value of an indicator is used as a pollution criterion. Since four pollution groups were being studied, four criteria were developed: (1) fish stock depletion indicator (FDI), (2) general indicator (GI), (3) toxicological indicator (TI), and (4) sanitation indicator (SI).

The system verifies the compatibility of the restrictions defined by bounds on performance indicator values. If no feasible solution can be found, the user must change the bound values. If they are compatible, the subsystem is ready to approximate the EPH. When the approximation is complete, the subsystem for the visual exploration of decision maps can be invoked (methods to solve problems with thousands of variables are described in the Appendix).

6.2. Initial display

At the very beginning of the visual analysis, the subsystem displays a picture containing one slice of the EPH. Two axis criteria are chosen automatically. Figure 4 depicts one slice depicting cost and sanitation indicators. Since this case study includes five criteria, the initial values of the other three criteria are fixed; their values are indicated on three scroll-bars under the picture of the slice. The efficiency frontier of the slice consists of two linear segments separated by a small kink. A click on any point of the picture reveals that point's coordinates. In Figure 4, a point on the efficient frontier was selected with coordinates 273.036 and 47.308.
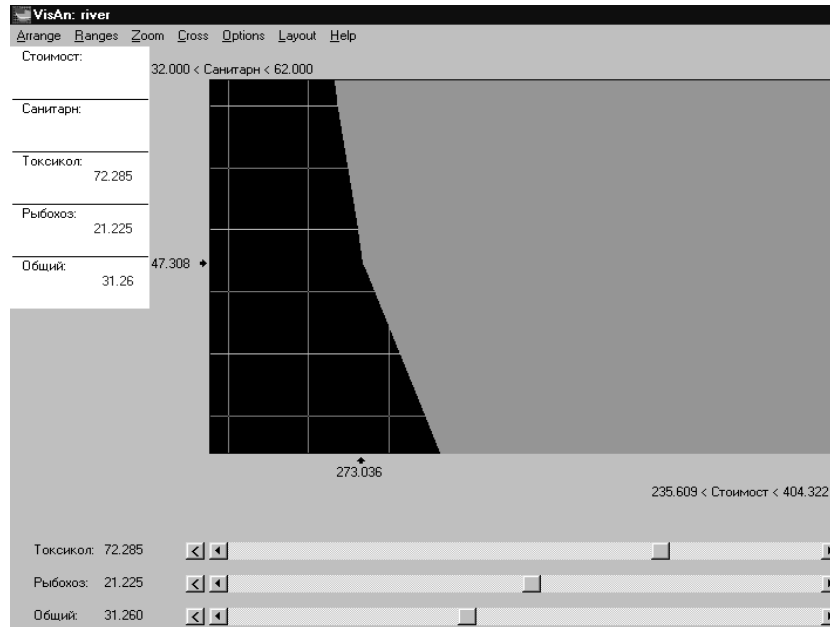

Figure 4. Efficient frontier for cost (horizontal axis) and sanitation (vertical axis) indicators.

It is possible to change the values of the fixed pollution criteria by moving the "thumbs" of the scroll bars. This will change the efficient frontier. Animation of the picture (automatic movement of the "thumbs") is also possible.

6.3. Decision map

The decision map in Figure 5 illustrates the efficient trade-offs between the GIs, SIs, and the cost. GI values are shown on the horizontal axis; SI values are shown on the vertical axis. Cost values are shown by means of different colours. The colour/cost correspondence is indicated in the palette beneath the decision map (in millions of 1988 rubles). The efficient frontiers for the GIs and SIs are the edges of the differently shaded areas.
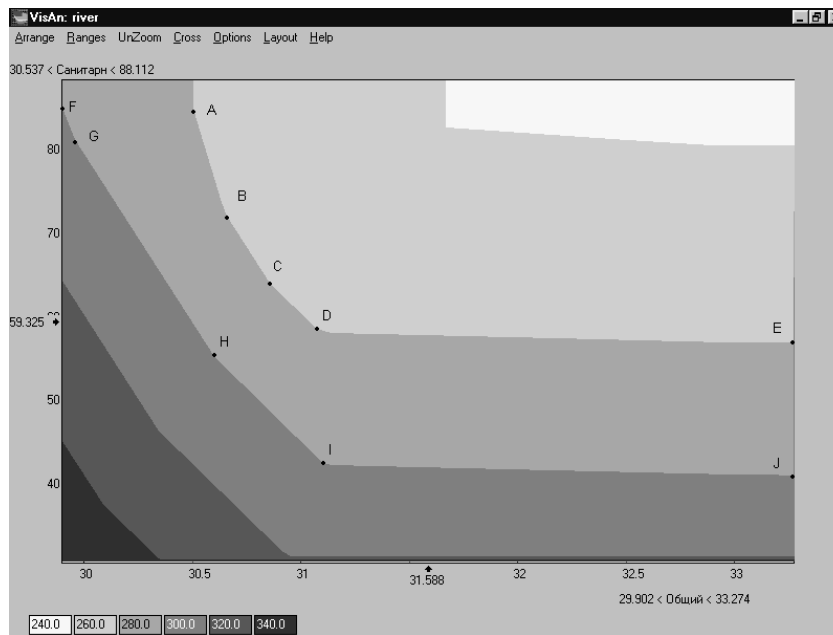

Figure 5. Efficient frontiers for general (horizontal axis) and sanitation (vertical axis) indicators.

The lightest shading corresponds to all the combinations of GI and SI values that are feasible at a minimum cost of 240 million rubles. With 240 million rubles spent on pollution abatement, the GI value cannot be less than about 31.6, and the SI value cannot be less than about 80.5. These values, as well as the coordinates of any other point given in the decision map, can be shown with a mouse click at the desired point.

The cost increase to 260 million rubles yields a larger set of feasible GI and SI values. The minimal feasible GI value decreases to 30.5 (point A). The SI value at point A cannot be less than 84.5. Alternatively, at 260 million rubles the SI value decreases

to 57 (point E). SI = 57 raises the GI value to 33.3. The shape of the frontier shows that with 260 million rubles available it is not possible to decrease the GI and SI to their minimal values simultaneously.

Points A, B, C, D, and E divide the efficient frontier for 260 million rubles into four segments. Trade-offs between the GI and SI change at points B, C, and D, so the unit decrement of the SI results in different increments of the GI at different points along the frontier. Depending on the preferences of the user, therefore, different points are likely to be chosen.

It is apparent that the cost increment results in an expansion of the area of efficient solutions. Every extra 20 million rubles expands the range of feasible GI and SI values. A GI value of 29.9 is not feasible at a cost of 260 million; a minimum of 280 million rubles is needed to reduce general pollution to that level. In this way, the decision map makes it easier to understand the relation between cost and water-quality improvements.

6.4. Matrix of decision maps

A user wishing to explore five criteria at once must use a matrix of decision maps (Figure 6). First, the user must arrange the criteria, that is, select two criteria whose values will correspond to the rows and columns of the matrix. In Figure 6, TI values correspond to the columns,and FDI values correspond to the rows. Any column is subject to a certain constraint imposed on the TI value (TI is not greater than…); these constraints are indicated above the columns. Any row is subject to a certain constraint imposed on the FDI value; these constraints are provided to the right of the rows.

A decision map located in a particular column and row is subject to constraints imposed on the TI and FDI values. There are only four rows and four columns in Figure 6, but it is possible to use as many rows and columns as the screen resolution permits. The constraints imposed on the TI and FDI values can be changed easily. The user can choose any single map in the matrix with a mouse click for more detailed exploration. If the user clicks on the decision map in Figure 6 which indicates TI = 100 and FDI = 26, the picture presented in Figure 5 will appear.

The top decision map in the third column represents a TI value no greater than 90 and a FDI value no greater than 26. Since the area that corresponds to the cost of 240 million rubles is absent from the decision map, these TI and FDI levels cannot be achieved at that cost. In this decision map, the cost areas begin at 260 million rubles; therefore, these levels of TI and FDI can be achieved only at that minimum cost.

In contrast, the bottom map in the third column represents an FDI value no greater than 16. The minimal cost in this case is 300 million rubles. In the decision map above that (FDI values no greater than 19.3), the minimal cost decreases to 280 million rubles. In the decision map that represents an FDI value no greater than 22.6, the minimal cost decreases to 260 million rubles and the efficiency frontiers that correspond to the greater cost increments move to the left. Therefore, a broader set of combinations of GI and SI values becomes feasible.

The efficient trade-offs among general and sanitation indicators also change from one row to another. This is additional information that may be important to a decision-maker.
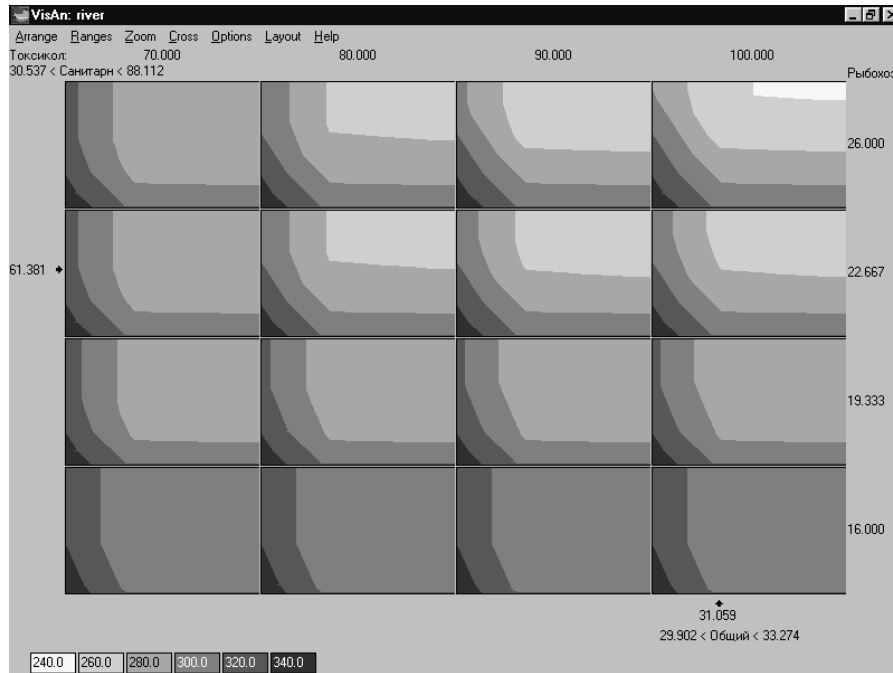
Figure 6. Matrix of decision maps.

The decision maps in the upper row of the matrix share a common FDI value of no greater than 26. The TI values change to no greater than 70, 80, 90, and 100 respectively. The minimal costs in these decision maps are 280 million rubles for TI no greater than 70; 260 million for TI no greater than 80; 260 million for TI no greater than 90; and 240 million for TI no greater than 100.

A more detailed analysis may entail a comparison of feasible values of axis criteria and the efficient frontier between them for different decision maps in the same row. Once again, these values can be found as coordinates of clicked points on a decision map. For example, let us compare the minimal GI values that are feasible at the minimal cost of 280 million rubles on the decision maps across the upper row. These values are:

GI = 30.7 for TI no greater than 70 (the related value of SI = 60.9);
GI = 30.4 for TI no greater than 80 (SI = 70.3);
GI = 30.0 for TI no greater than 90 (SI = 79.1); and
GI = 29.9 for TI no greater than 100 (SI = 84.5).

Since the approximation of the EPH is conducted in advance, decision maps are displayed relatively quickly. An updated matrix of decision maps is displayed a few seconds after the values of the fourth and the fifth criteria have been changed. It is also possible to change quickly the list of the three criteria for which trade-offs are to be displayed.

When comparing decision maps in the matrix for properly chosen fixed values of the fourth and fifth criteria, the user can understand the influence of these two criteria on efficient trade-offs among the first three. Hence, the efficient trade-offs among all five criteria displayed by decision map matrices, are shown in an interactive context.

In a case where six or seven criteria are necessary, more scroll-bars can be provided under the decision map matrix. The user can change the values of these criteria by moving the "thumbs". The matrix of decision maps may be animated. This time-consuming animation is possible because of the prior approximation of the EPH (The FGM/IDM software can be obtained at http://www.ccas.ru/mmes/mmeda/soft/).

6.5. Feasible goal and strategies

Users have full freedom to choose the point in the criteria space with regard to the efficient frontier. By choosing one of the decision maps in a matrix, the user identifies the values of the row and the column criteria. After that, the procedure identifies the feasible values for the other three criteria. The particular values are displayed for any decision selected on the map. The subsystem for decision computing then bases a strategy for water quality improvement on the identified feasible goal. The strategy can then be displayed in maps generated by a GIS module of the system (Figure 7). Monitoring stations are marked on the map with numbered icons.

To receive information about investment allocation along a certain segment of the river and pollution concentrations in its related monitoring station, the user must click on the appropriate icon.
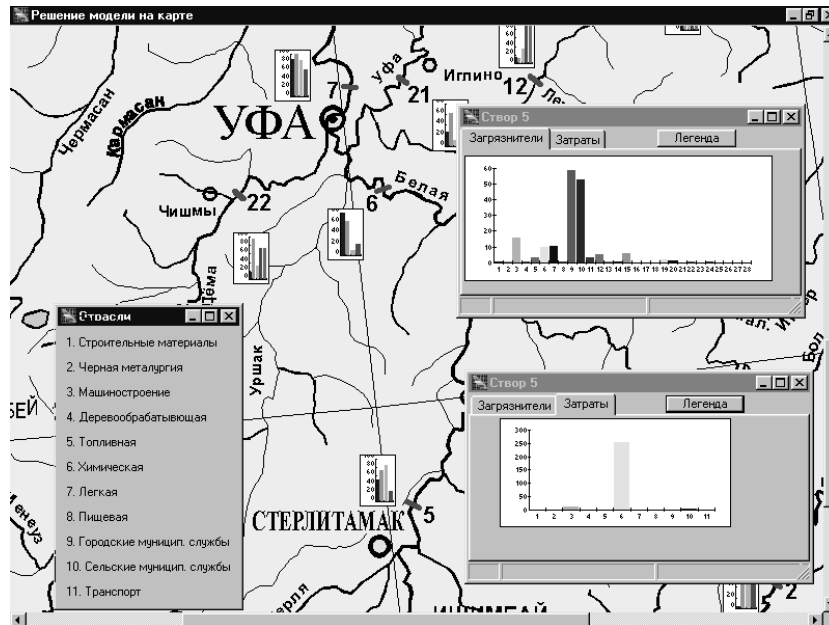

Figure 7. A segment of the river basin map generated by a GIS module.

In this case, the icon for segment number 5 was selected, which generated a display of additional windows. The upper window shows the concentrations of 28 pollutants in the monitoring station (after the discharge treatment facilities are completed). The lower window displays information on investment allocation; in this case, it illustrates the funds invested in 11 industries. The window on the left lists the industries themselves (1. construction materials production; 2. ferrous metallurgy; 3. machinery construction; 4. woodworking industry; 5. fuel industry; 6. chemical industry, and others).

The DSS also gives the user an opportunity to study the strategy from another point of view. Users are provided with aggregated information on all the values at all of the monitoring stations. Diagrams are used to describe how pollution indicators change from one monitoring station to another.

6.6. Applications

The next step is the installation of the DSS in the regional centers located in the Volga River basin. The system will help regional authorities to develop strategies that are profitable for them and acceptable to the Ministry of Natural Resources, as well as to the other regions. In this form, the system will support environmental negotiations.

In Thiessen and Loucks (1992), a negotiation support system can be categorized according to its function as a negotiation preparation system or as a negotiation process support system. The former supports prenegotiation strategic planning, while the latter facilitates the negotiation processes. The above application of the DSS incorporates its function as a negotiation preparation system. In this form, it cannot be used to support a negotiation process.

7. The FGM/IDM technique on the Web

Within the framework of the FGM/IDM technique, interaction between the user and the computer is quite simple. For this reason, it can be implemented on computer networks such as the Internet. Now, new and exciting opportunities that did not exist earlier can be provided for network users. In particular, FGM/IDM-based Internet resources can provide direct access to information on a myriad of possible decision alternatives. This access is important in various fields; however, from the societal point of view, its most important asset is the ability of the FGM/IDM technique to support an Internet-based search for preferred strategies that may solve important public problems. The technique also can help to obtain unbiased information on these solution options; in other words, information that is not screened by experts and other mediators (Williams and Pavlik, 1994). Moreover, the FGM/IDM technique supports an independent search for a preferred strategy to help ordinary people be actively involved in public discussions on matters that affect them directly.

A prototype Internet resource (http://www.ccas.ru/mmes/mmeda/resource) demonstrates this idea on the basis of a problem considered in detail in Lotov *et al.,* (1997). This prototype allows the user to study the problem of water quality planning in a region with developed agricultural production. If the agricultural production increases,

it may spoil the water quality in a lake. Therefore, reasonable strategies for agricultural production and water management must be developed.

The efficient trade-offs among several criteria are displayed in three premade decision maps. The user can choose an appropriate decision map and identify a preferable feasible goal; all that is required for the user to develop a preferred feasible strategy is Internet access and a Web browser.

The Internet resource described here is only a prototype of the future Internet resources that will use the FGM/IDM technique to its full capacity. The application of resources of this kind will help to develop strategies regarding various public problems, such as regulating a national economy or the resolution of local, regional, national, or even global environmental problems. It will help people to understand the complications that stakeholders face in the process of trying to solve public problems and it may prompt them to try to influence the decision-making process.

8. Appendix: The FGM/IDM technique

In the framework of the FGM/IDM technique, the generation of efficient frontiers and the screening of feasible decisions are based on the algorithms of a universal mathematical approach called the generalized reachable sets (GRS) method (Lotov, 1973). The GRS method is used to approximate and display a variety of attainable output vectors for a large (or infinite) variety of possible input values. In decision problems, the computational algorithms of the GRS method transform feasible decisions into feasible combinations of criterion values — the FSCS or its EPH.

8.1. The EPH of the FSCS

Let the decision variable x be an element of the decision space W (e.g., of finite-dimensional space R^n; in this case, decision vectors x are considered). Let the set of feasible decisions X from the space W be given. Let the criterion vector y be an element of linear finite-dimensional space R^m. If it is assumed that criterion vectors y are related to decisions by a given mapping $f: W \rightarrow R^m$, then the FSCS is defined as the variety of criterion vectors that are attainable if all the feasible decisions are used:

$$Y = \left\{ y \in R^m : y = f(x), x \in X \right\}.$$

Let us suppose that a user is interested in the increment of the criterion values y. In this case, a criterion point y' dominates (is better than) another criterion point y, if $y' \geq y$ (i.e. $y_i' \geq y_i$ for $i=1,2,...,m$) and $y' \neq y$. Then, the nondominated frontier of the FSCS is defined as the variety of nondominated points of $y \in Y$, or points for which the dominating points do not exist in Y (the set of dominating points is empty):

$$P(Y) = \left\{ y \in Y : \left\{ y' \in Y : y' \geq y, y' \neq y \right\} = \varnothing \right\}.$$

The nondominated frontier is often identified as the Edgeworth-Pareto set.

Another set in criterion space is used to determine a nondominated frontier of the FSCS that has a simpler structure than the FSCS, but has the same nondominated frontier. This set is called the EPH of the FSCS. EPH is defined as

$$Y* = Y + (- R_+^m),$$

where R_+^m is the nonnegative cone of R^m. Both Y and $Y*$ have the same nondominated frontier:

$$P(Y*) = P(Y).$$

Note that the dominated frontiers disappear in $Y*$.

Decision maps are calculated as collections of two-dimensional (two-criterion) slices (cross-sections) of the EPH. Let u denote the values of two selected criteria and $z*$ denote the fixed values of the remaining criteria. Then, a two-dimensional slice of the set $Y*$ related to $z*$ is defined as

$$G(Y*, z*) = \{u : (u, z*) \in Y *\}.$$

It is important to note that a slice of the EPH contains all combinations of the values of the two criteria that are feasible if the values of the remaining criteria are not worse than $z*$.

In the IDM/FGM technique, decision maps help to identify a preferable feasible goal. Once identified, y' is regarded as the "reference point" (Wierzbicki, 1981), that is, an efficient decision is obtained by solving the following optimization problem:

$$\min_{1 \le j \le m}(y_j - y_j') + \sum_{j=1}^{m}\left\{\varepsilon_j(y_j - y_j')\right\} \Rightarrow \max,$$

for $y = f(x), x \in X,$

where $\varepsilon_1, \ldots, \varepsilon_m$ are small positive parameters. Since the goal is close to the EPH, the efficient decision results in criterion values that are close to the goal.

8.2. Approximation of the EPH

Decision maps are computed and depicted quickly with the FGM/IDM technique, since the EPH is approximated in advance. If the EPH is a convex set, its approximation of the EPH is based on iterative methods combining convolution methods (Fourier, 1826) and optimization techniques. The EPH is approximated by the sum of the cone (- R_+^m) and a polytope approximating the FSCS. A short description of the convolution-based methods is provided in Lotov (1996). If the EPH is a nonconvex set, an approximation of the FSCS and its EPH is more complicated, but is nonetheless more possible (Kamenev and Kondratiev, 1992; Bushenkov *et al.*, 1995).

Linear finite-dimensional models are usually applied in the integrated assessment of environmental problems. The space W is a linear space R^n, the set X is a polytope of R^n, and the mapping $f: R^n \rightarrow R^m$ is linear. Usually, the set X is specified by a linear constraint system:

$$X = \{x \in R^n: H\,x \le h\},$$

where H and h are a given matrix and vector, respectively. The linear mapping

$$f{:}R^n{\to}R^m$$

is specified by a matrix F, such that $f(x) = F\,x$.

The FSCS

$$Y = f(X) = \{\, y \in R^m : y = F\,x,\; H\,x \le h\,\}$$

can be approximated by a polytope Q, constructed in the form

$$Q = \{\, y \in R^m : Dy \le d\},$$

where D is a matrix and d is a vector to be computed.

8.3 Approximation of the FSCS

As stated above, the EPH is approximated by the sum of the cone ($- R_+^m$) and a polytope approximating the FSCS. Methods for the approximation of the FSCS and its EPH are closely related (details are provided in Chernykh, 1995).

Let us consider the general problem of the approximation of a convex compact body C from R^m. It is assumed that the values of the support function of the body C can be computed, which is defined by the formula on the unit sphere of directions

$$g_C (u) = \max \{{<}u, y{>}: y \in C\},$$

where ${<}.,.{>}$ is the scalar product. In the case of the FSCS for a linear problem, the value of the support function can be determined by solving a linear programing problem. Here, two methods are described for the approximation of convex compact bodies. Both of them are iterative; they allow a sequence of convex polytopes P^0, P^1, ..., P^k, ... with an increasing number of vertices. Every polytope in the sequence is based on the previous one, using evaluations of the support function for the approximated body C. These methods construct a sequence of convex polytopes $P^k \in R^m$ such that

$$\lim_{k\to\infty} \delta(P^k,C) = 0\,,$$

where $\delta (.,.)$ is the Hausdorff distance between any two compact sets from R^m:

$$\delta(C_1, C_2) = \max \{\, \sup \{d(x, C_2): x \in C_1 \},\; \sup \{d(x, C_1): x \in C_2 \}\,\},$$

and $d(.,.)$ is the distance in R^m.

Note that these are methods to approximate the body C to the greatest degree of accuracy;.that is an important difference when these methods are compared with those that use a simplex or ellipsoid to estimate a body. Obtaining this degree of precision,

however, comes at a high price: the complexity of an approximating polytope increases rapidly if the accuracy of approximation and the dimension m increase. Therefore, methods are needed that have an optimal rate of convergence and an optimal complexity of approximating polytopes.

To introduce the concepts of optimal rate of convergence and of optimal complexity of approximating polytopes, it is necessary to consider a "standard" polytope that provides the best approximation of the convex compact body C. Let $P(C)$ denote the class of convex polytopes whose vertices belong to the boundary of C. Let $P_N(C)$ denote the subclass of polytopes with the number of vertices no greater than N. A classical tenet in the theory of convex sets dictates that there always exists a polytope $P_N \in P_N(C)$, such that

$$\delta(C, P_N) = \delta(C, P_N(C)) = \inf\{\delta(C, P) : P \in P_N(C)\}.$$

This polytope P_N is called the best approximation polytope; thus

$$\lim_{k \to \infty} \delta(C, P_N) = 0.$$

For any convex compact body C with twice continuously differentiable boundary and positive principle curvatures, there exist positive constants k_C, and K^C such that

$$k_C N^{\frac{2}{1-m}} \leq \delta(C, P_N) \leq K^C N^{\frac{2}{1-m}}.$$

Incidentally, this formula shows that a body with a smooth boundary may be approximated by polytopes with reasonable accuracy only if $m<8$. It is interesting to compare the sequence of the best approximation polytopes generated by a particular approximation method to the sequence of polytopes of the best approximation itself.

Two iterative methods for the approximation of convex bodies by polytopes are described here. The first one, called the Estimate Refinement (ER) method, is a typical example of an iterative method.

8.4. The Estimate Refinement method

Let $U(P)$ be the finite set of outer unit normals to the facets of a polytope P. The set $U(P)$ is defined if the polytope P is specified as the solution set of a system of linear inequalities.

Following is a description of the $(k+1)$-th iteration of the ER method. Prior to the iteration,

$$P^k \in P(C)$$

should be constructed as the solution set of a linear inequalities system; then the $(k+1)$-th iteration will consist of two steps.

Step 1. Find the element $u^* \in U(P^k)$ that provides the maximal difference between the values of the support function of the body C and the support function of the polytope P^k:

$$g_C(u^*) - g_{P^k}(u^*) = \max\{ (g_C(u) - g_{P^k}(u)) : u \in U(P^k) \}$$
.

Find a point p^* from the boundary of the body C, such that $<u^*, p^*> = g_C(u^*)$.
   Step 2. Construct the convex hull of the point p^* and the polytope P^k:

$$P^{k+1} = \text{conv}\{p^*, P^k\}$$

in the form of the solution set of a linear inequalities system.

It is assumed that the initial polytope $P^0 \in P(C)$ has been specified prior to the application of the method. The construction of the initial polytope is discussed in Chernykh (1992).

It was demonstrated in Kamenev (1994) that for bodies with a smooth boundary, the ER-generated sequence of polytopes P^k has the same rate of convergence as the best approximating polytopes P_N, $N = m+1, m+2, m+3,....$ Therefore, the ER method is optimal with respect to the number of vertices. Other important properties of the ER method are provided in Kamenev (1994).

In real-life environmental problems (such as those studied in this paper), the set X belongs to a space R^n of a high dimension, where the value of n is several thousand. The evaluation of the support function is related to the solution of a linear programing problem; with thousands of variables, this can be a lengthy process. Methods that have been developed until recently (including the ER method) require multiple evaluations of the support function during an iteration. Thus, it became important to develop a new method that would be optimal with respect to the number of evaluations of the support function. Such a method was proposed by Kamenev (1996), called the Converging Polytopes (CP) method.

8.5. The Converging Polytopes method

In the framework of the CP method, a body C is approximated using two polytope sequences. One consists of inscribed polytopes with the tops belonging to the body boundary, and the other consists of circumscribed polytopes. These polytope sequences come closer and closer together during the process of approximation.

Let $Q(C)$ denote a set of convex polytopes whose facets touch the boundary of the set. Once $P^k \in P(C)$ and $Q^k \in Q(C)$ are constructed, the $(k+1)$-th iteration will involve the following two steps:
   Step 1. Find the element $u^* 0 U(P^k)$ that provides the maximal difference between the support functions of the inscribed and the circumscribed polytopes:

$$g_{Q^k}(u^*) - g_{P^k}(u^*) = \max\{(g_{Q^k}(u) - g_{P^k}(u)) : u \in U(P^k)\}.$$

Find a point p^* from the boundary of the body such that $<u^*, p^*> = g_C(u^*)$.
   Step 2. Construct the convex hull of the point p^* and the polytope P^k:

$$P^{k+1} = \text{conv}\{p^*, P^k\}$$

in the form of the solution set of a linear inequalities system. Put

$$Q^{k+1} = Q^k \cap \{y \in R^m : <u^*, y> \leq g_C(u^*)\}.$$

It is clear that only one evaluation of the support function is required for each iteration. Therefore, the CP method requires a smaller number of evaluations of the support function than vertex selection methods do.

An experimental comparison of the CP and ER methods was performed, related to the approximation of various ellipsoids of different dimensions. The study showed that the CP method generates polytopes that have approximately the same number of vertices for the same precision of approximation as the polytopes generated by the ER method. This is a significant result, since the CP method requires a substantially smaller number of evaluations of the support function in order to obtain a polytope with the same number of vertices.

The application of the CP method makes it possible to construct the EPH in a reasonable amount of time, in spite of a lenghty evaluation of the support function. In turn, this facilitates the construction of a DSS for water quality planning in large rivers.

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