Interactive Decision Maps, with an Example Illustrating Ocean Waste Management Decisions

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Abstract. Interactive Decision Maps are introduced and illustrated with an ocean waste disposal example, requiring difficult pollution-cost tradeoffs. Interactive Decision Maps are a tool for quickly displaying various decision maps for three or more decision-relevant criteria. They are based on Generalized Reachable Sets (GRS) approach developed in Russia. Animation of decision maps is also possible. Integration of Interactive Decision Maps with Pareto Race, a free search Multiple Objective Linear Programming procedure, is proposed.
Keywords: Multiple Objective Linear Programming, Nondominated Set, Ocean Waste Management

1. Introduction

Decision maps are a well known, but rarely used multiple criteria decision support tool developed for the case of three choice criteria (see, for example, Haimes et al., 1990, Figure 4.2). A family of bi-criteria cross-sections of Pareto-optimal frontier, assuming a fixed value for the third criterion is depicted. The Interactive Decision Maps (IDM) technique is a tool for fast display of modified decision maps where bi-criteria Pareto-optimal frontiers are depicted while the third criterion is not permitted to become worse than a prespecified value. A decision map which looks like a geographical map helps to understand the tradeoffs among the criteria. In the case of more than three criteria, various decision maps are displayed (upon request). Animation of decision maps, i.e. display of automatically generated sequences of maps, is also possible.

The IDM technique seeks to help decision makers to identify a most preferred feasible combination of criterion values with a simple click of the mouse on the appropriate decision map. When such a combination has been identified, a point in the decision variable space which leads to this most preferred point in the criterion space is computed (the Feasible Goals Method (FGM) introduced by Lotov (1973, 1984)).

In this paper we reconsider the old problem of choosing sewage sludge disposal sites in the New York Bight with the help of the IDM/FGM technique. This problem was considered by Wallenius, Leschine, and Verdini (1987) and by Leschine, Wallenius and Verdini (1992). In conclusion, the integration of Pareto Race (Korhonen and Wallenius, 1988), a free search Multiple Objective Linear Programming procedure, with the IDM technique is proposed. An appendix contains a mathematical formulation of the IDM/FGM technique.

2.Sewage Sludge Disposal Problem

Contamination of the New York Bight has been a concern of the Environmental Protection Agency (EPA), and the neighboring municipalities for many years. Concern for water quality in the Bight region is long standing, particularly for waters in the inner portion of the Bight. Highly publicized pollution-related episodes which have occurred over the past decades have had a lasting impact on public opinion. Being concerned about the contamination of the inner Bight region, the EPA ordered in 1985 New York City and the remaining users of the inner Bight region to begin shifting their dumping operations to the 106-mile site. In this study we reexamine, following Wallenius et al. (1987) and Leschine et al. (1992), the EPA decision in a way which permits simultaneous multi-site dumping.

Three alternative disposal sites were considered in the model: the 12-mile site, the 60-mile site, and the 106-mile site. We assumed that a combination of the above sites was a possibility, such that all three sites could be used at the same time in different portions. In the model all sludge was assumed to be produced in New York City (where 52% is produced), New Jersey (41%), and Long Island (7%). Production of sludge was assumed to be constant from year to year. Two types of vessels were used for the transportation of the sludge: towed barges, and self-propelled barges. The constraint set of the model contained four parts:

1) constraints to ensure dumping of all generated sludge;
2) constraints of annual dumping capacity of barges;
3) definitional constraints of amount dumped at each site;
4) Markov constraints to model the ocean's assimilative capacity.

The following three criteria were used to evaluate different sludge disposal strategies:

- total cost of sludge disposal operation (millions of US$);
- pollution level at inshore monitoring station (pollution concentration, in percent to a given value);
- a given value pollution level at offshore monitoring station (pollution concentration, in percent to).

The decision variables included the number of self-propelled/towed barge trips from source (NY, NJ, LI) to site (12-, 60-, 106-mile sites). A formal description of the model is given in Leschine et al. (1992).

3. Interactive Decision Maps

The IDM technique is a particular form of the Generalized Reachable Sets (GRS) method (Lotov, 1973, 1984; see also Lieberman, 1991). The GRS method was developed for the exploration of nonclosed mathematical models. It consists of constructing and displaying the set of attainable output (criterion) vectors for a given feasible combination of input variables. In the MCDM context, the GRS method provides an opportunity to transform a decision problem from the space of decision variables into the criterion space. To be precise, starting with the set of feasible strategies, we construct the set of feasible combinations of criterion values. Aggregate information is provided in the form of various decision maps which are displayed on request.

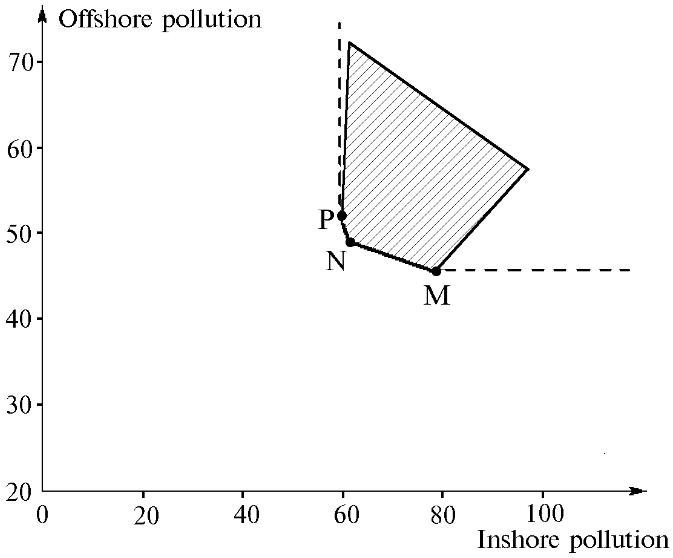

Figure 1: Feasible Set in Criterion Space

Let us consider the problem from the previous section. To begin with, let us fix the total cost. Then all feasible values of inshore pollution and of offshore pollution can be visualized on the computer screen in the form of an image (Figure 1). Since it is preferable to decrease both inshore and offshore pollution (ceteris paribus), we are interested in the "south-western" frontier of the image (the Pareto-optimal frontier). The Pareto-optimal frontier contains those combinations of inshore and offshore pollution levels which have the following property: to decrease one objective (say, offshore pollution) one needs to increase the other (inshore pollution). The form of the Pareto-optimal frontier shows how large of a drop in the offshore pollution is connected with a certain increment in the inshore pollution, and vice versa. In Figure 1 a small decrement in the offshore pollution requires a substantial increment in the inshore pollution near point "M". And correspondingly, near point “P” just a small rise in the inshore pollution results in a sharp decrement in offshore pollution.

Note that the total cost is fixed in Figure 1. To vary the value of the cost criterion, a decision map, consisting of a family of Pareto-optimal frontiers is constructed. The display of such maps in the framework of the IDM technique is based on approximating the Edgeworth-Pareto Hull (EPH) of the Feasible Set in Criterion Space (FSCS), i.e. of the FSCS augmented with all dominated criterion points. The shaded region in Figure 1 is an example of an FSCS for two criteria. The frontier of the EPH is depicted by dashed lines. Note that the EPH has the same Pareto-optimal frontier as the FSCS. The same is true for any number of criteria: the Pareto-optimal frontiers of the FSCS and its EPH coincide.

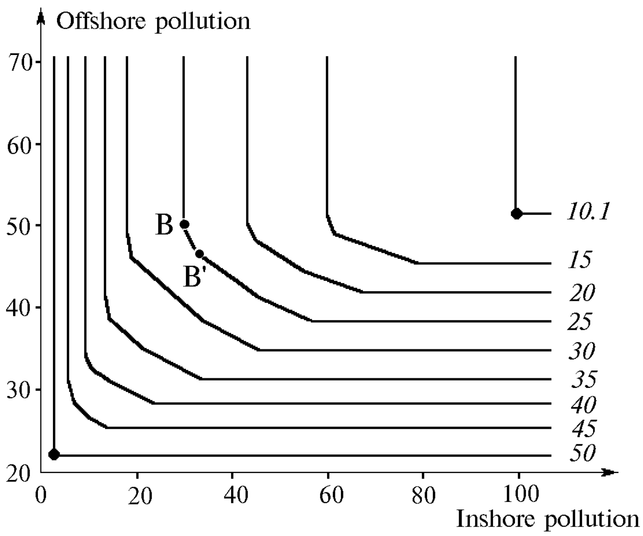

Figure 2: A Decision Map for Fixed Costs

Decision maps are constructed as collections of two-dimensional slices of the EPH. A decision map containing the Pareto-optimal frontiers between inshore and offshore pollution related to different values of total cost is shown in Figure 2. With the help of the decision map, one can easily understand the relation between an increment in the total cost and the improvement of the environment (i.e., a reduction in the inshore and/or the offshore pollution). One can easily obtain different decision maps displaying the tradeoffs “inshore pollution vs. cost” and “offshore pollution vs. cost”.

The approximation of the EPH is constructed in advance in the framework of the IDM technique. This is the main mathematical and computational problem which has been solved during the development of the IDM technique. Three groups of methods for the approximation of the EPH were developed. Methods of the first group are based on direct application of the classic Fourier convolution of systems of linear inequalities and may be used in the case of linear models with a relatively small number of decision variables (see Lotov, 1996). The second group of methods can be applied for linear systems with a large number of decision variables (several thousands), but a relatively small number of criteria (three to six). The basic idea of the methods of the second group consists of constructing a sequence of polytopes iteratively approximating the FSCS. Such methods extend the idea of the NISE method earlier proposed for two criteria problems (Cohon, 1978). Sequences of polytopes are constructed on the basis of a combination of the Fourier convolution and optimization techniques. It has been proven that the approximation methods produce optimal sequences of polytopes. The methods of the third group are related to approximating the FSCS and the EPH for nonlinear models. Details and references are given in Bushenkov et al. (1995).

The IDM technique displays collections of two-dimensional slices of the EPH in the form of decision maps. If more than three (say, five) criteria are incorporated, one needs to impose constraints on the values of the additional criteria to obtain a decision map. This can be done manually by using scroll-bars. Animation of a decision map may be performed by displaying a sequence of maps related to a sequence of constraints generated automatically. Moreover, one can display a matrix of decision maps corresponding to a collection of constraints imposed on the values of the additional criteria. These constraints may be chosen manually or generated automatically. A software system FEASIBLE GOALS for MS WINDOWS 3.1, implementing the IDM/FGM methodology has been developed.

4. Further Analysis of the Waste Disposal Problem

Let us take a closer look at Figure 2. In Figure 2 one can see the decision map in the “inshore pollution — offshore pollution” space, while the cost is changing from a minimal US $10.1 million to the maximal US $50 million. The Pareto-optimal frontiers in Figure 2 were drawn with heavy lines. They have the following important feature: they are kinked, except for the $10.1 million and $50 million frontiers. (Actually, the Pareto-optimal frontiers related to $10.1 million and $50 million consist of just one point.) The kink depends upon the cost. Above the kink, the tradeoff between inshore and offshore pollution is quite different from the tradeoff below it.

Compare the distances between pairs of Pareto-optimal frontiers related to different costs. The $10.1 million and $15 million frontiers are quite apart, while the distance between the $15 million and $20 million frontiers is obviously smaller. This means that the extra $5 million investment has much more impact if the cost equals $10.1 million rather than $15 million.

The above phenomenon is explored from another angle in Figure 3 where an alternative decision map is displayed. The “cost — inshore pollution” tradeoff curves are given for several values of offshore pollution, ranging from 25% to 55% of its maximum value. One can see that every frontier contains a kink where the tradeoff changes drastically. If the offshore pollution equals 55%, the kink occurs at point A. Note that point A also belongs to another frontier which corresponds to 50% offshore pollution.

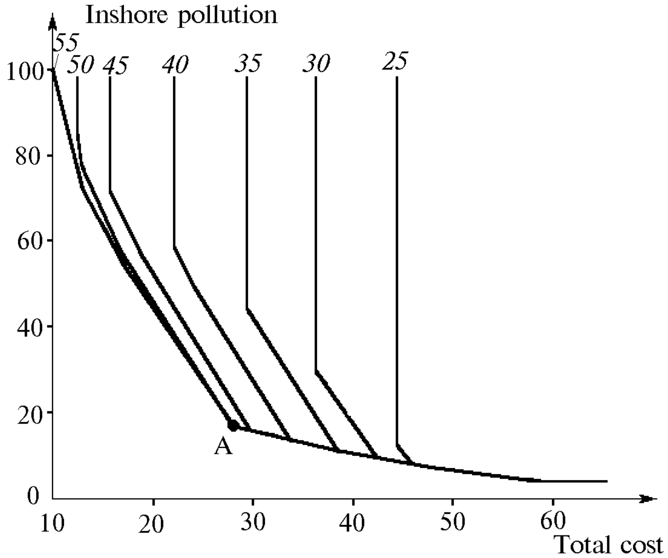

Figure 3: Total Cost Vs. Inshore Pollution

Figure 4 provides a close-up of the inshore pollution vs. total cost tradeoff, when the values of offshore pollution vary between 43% and 55%. It is interesting to

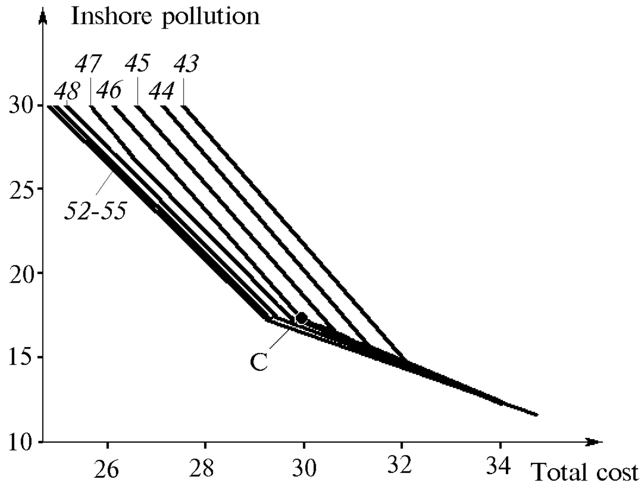

Figure 4: Inshore Pollution Vs. Cost: A Close-Up

note that if the offshore pollution exceeds 46%, further growth in it is practically useless: additional offshore pollution moves the frontier only marginally downwards. For this reason, the following combination of criterion values (point C) may be of interest:

- cost = $30 million,
- inshore pollution = 17.5%,
- offshore pollution = 46%.

Suppose point C has been chosen as the decision maker's most preferred point. The associated decision provided by the computer is the following: transport all the waste to the 60-mile site.

5. Integrating IDM and Pareto Race: A Suggestion

Pareto Race was developed by Korhonen and Wallenius in the late 80's (Korhonen and Wallenius, 1988) as a dynamic and visual "free-search" type of interactive procedure for Multiple Objective Linear Programming. The procedure enables a decision maker to freely search any part of the Pareto-optimal frontier by controlling the speed and direction of motion. The criterion values are represented numerically and as bar graphs on a display. The keyboard controls include gears, an accelerator, brakes, and a steering mechanism. Using Pareto Race to explore the Pareto-optimal frontier resembles driving an automobile. The driver does not, however, have a map of the terrain that he/she explores. The decision maker usually has an idea in which direction he/she would like to move. But without a map, he/she does not always know if it is possible or worthwhile to move in a certain direction. The IDM could provide this missing map and guide the user through the search.

We describe one possible design for integrating Pareto Race and IDM. The main screen of the system is split in three main parts (Figure 5). The left/middle upper part is related to the IDM, where the decision map is displayed. As one can see, the given decision map is actually a part of Figure 2 augmented with the current point (the cross) and an additional slice related to the current point (depicted by a dashed line). The current values of inshore pollution and offshore pollution are indicated by the position of the cross. At the same time, the color (in the paper—the shading) of the field containing the cross informs the decision maker about the cost interval. The diagram relating the color of the frontier (the band) to the value of total cost is shown under the decision map.

The dashed tradeoff curve which moves together with the cross over the decision map informs the decision maker about the tradeoff among inshore and offshore pollution and helps him/her decide whether it is reasonable/worthwhile to move along the chosen direction. The IDM control window is provided in the top-leftmost corner. All opportunities of the IDM technique are available. In particular, one can request another decision map related to the problem or change the number of Pareto-optimal frontiers. Moreover, a zoomed portion of the decision map may be displayed. In case of more than three (say five) criteria, one can obtain various matrices of decision maps, animated decision maps, etc. In the bottom part of the screen, the current criterion values are depicted in the form of horizontal bars used in Pareto Race. The right-hand side of the screen is a fictitious Pareto Race control panel supplemented with an additional PAUSE/GO window.

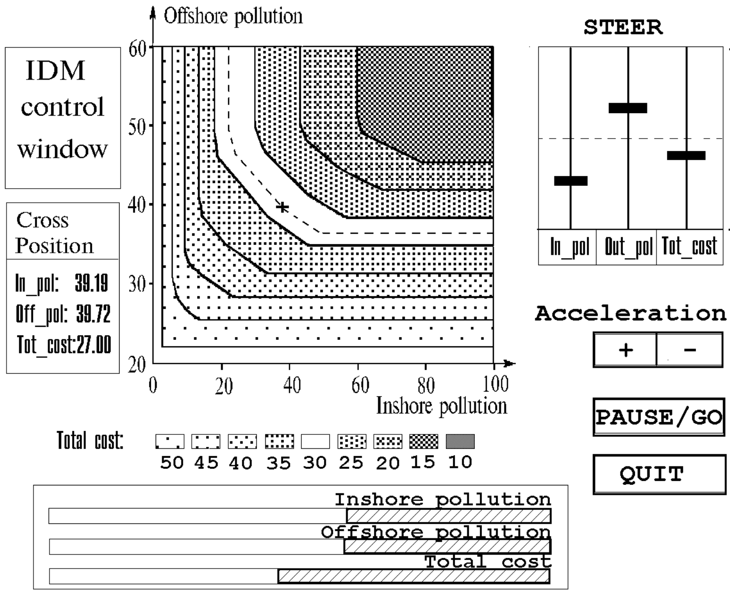

Figure 5: Integrating Pareto Race and IDM: A Suggestion

The STEER window provides the opportunity to control the direction of motion. It performs like a standard audio equalizer. 'Zero level' means that we do not want to change the search direction for this particular criterion. 'Plus one' corresponds to a maximal improvement in the underlying criterion and 'minus one' to its opposite. The ACCELERATION window provides an opportunity to increase or decrease speed. The PAUSE/GO window helps to pause the process to explore the decision map more carefully.

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Appendix: A Mathematical Introduction to the IDM Technique

Let the decision variable x be an element of the decision space W. Let the set of feasible combinations of values of decision variables be $X \subset W$. Let the criterion vector y be an element of the linear finite-dimensional space R^m. We assume that the criterion vectors are related to decisions by a given mapping $f: W \to R^m$. Then the feasible set in the criterion space (FSCS) is defined as follows:

$$Y = \{y \in R^m : y = f(x), x \in X\}.$$

Let us suppose that we are interested, without loss of generality, in minimizing the criterion values. Then the Edgeworth-Pareto Hull (EPH) of the FSCS is defined as $Y^* = Y + R_+^m$, where R_+^m is the nonnegative cone of R^m. The Edgeworth-Pareto frontier of the FSCS is defined as

$$P(Y) = \{y \in Y: \{y' \in Y: y' \le y, y' \ne y\} = \varnothing\}.$$

It is clear that $P(Y^*) = P(Y)$.

The IDM technique consists of constructing the EPH and interactively displaying it in the form of decision maps. Constructing set Y^* is based on approximating it by the sum of a simple body approximating set Y and the cone R_+^m. In case of a convex set Y^*, polytopes are used as the approximating bodies. Nonconvex sets Y^* are approximated as the sum of the cone R_+^m and a set of cubes.

The EPH is displayed in the form of decision maps, i.e. collections of its two-dimensional slices. Let $I^0 = \{1, 2, \dots, m\}$. Also let $I \subset I^0$, $|I| = 2$. Finally, let $I^* = I^0 \setminus I$. Let us denote by $R(I)$ the criterion subspace with arguments from I. Then, by a two-dimensional slice of a set $V \subset R^m$ related to $z \in R(I^*)$ we mean the set

$$G(V, z) = \{u \in R(I) : (u, z) \in V\}.$$

It is important to note that in the case of the EPH, i.e. set Y^*, a slice of it contains the combinations of the values of the criteria from I which are feasible if the values of criteria from I^* aren't worse than z. Since the EPH is constructed in the framework of the IDM technique in advance, a collection of its two-dimensional slices can be depicted quite fast. Efficient algorithms for constructing the two-dimensional slices were developed in Chernykh and Kamenev (1993).

Once a most preferred feasible point in the criterion space has been identified, it is possible to obtain the decision which will lead to this point. If an appropriate feasible point in the criterion space yr has been identified, it is regarded as the 'reference point' (Wierzbicki, 1981). An efficient decision is obtained by solving the following optimization problem

$$\min_{1 \le j \le m}\left(y'_j - y_j\right) + \sum_{j=1}^{m}\left\{\varepsilon_j\left(y'_j - y_j\right)\right\} \Rightarrow \max,$$

while $y = f(x)$, $x \in X$, where $\varepsilon_1, \dots, \varepsilon_m$ are small positive parameters.

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